647510 **BSCmaMJ202x-1C** **Seat No :_____**

B.Sc. Semester - 1 (NEP-2020) Examination
OCT/NOV-2025 (As Per Syllabus Year June-2023)
MATH: Calculus and Solid Geometry (Major-2)

Time: 2:00 Hours **Marks:50**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Que.1(A) Answer any one. (05)

(1) State and prove Lagrange's mean value theorem and explain its geometrical interpretation.

(2) Show that: $\log(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \cdots$ where $-1 < x < 1$.

Que.1(B) Answer any one. (05)

(1) Separate the intervals in which the function $f(x) = 2x^3 - 9x^2 + 12x - 5$ is increasing or decreasing.

(2) State Maclaurin's infinite series and show that: $sinx = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots$

Que.2(A) Answer any one. (05)

(1) State and prove L'Hospital's rule for $\frac{0}{0}$ form.

(2) Evaluate: $\lim_{x \to \frac{\pi}{2}} \left(cot^2 x - \frac{1}{x^2}\right)$.

Que.2(B) Answer any one. (05)

(1) Evaluate: $\lim_{x \to 0} \frac{1+sinx-cosx+\log(1-x)}{xtan^2 x}$

(2) Evaluate: (i) $\lim_{x \to 0} \left(\frac{1}{x^2} - \frac{1}{sin^2 x}\right)$

(ii) $\lim_{x \to \frac{\pi}{2}} (sec^2 x)^{cot^2 x}, -\frac{\pi}{2} < x < \frac{\pi}{2}$.

Que.3(A) Answer any one. (05)

(1) Find the center and radius of the circle that is obtained as the intersection of the plane $x + 2y + 2z = 15$ and the sphere $x^2 + y^2 + z^2 - 2y - 4z - 20 = 0$.

(2) Find the equation of the tangent plane at any point (α, β, γ) of the sphere $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$.

Que.3(B) Answer any one. (05)

(1) Find the equation of the sphere which touches the sphere $x^2 + y^2 + z^2 - x + 3y + 2z - 3 = 0$ at the point $(1,1,-1)$ and passes through the origin.

(2) Prove that the spheres $|\bar{r}|^2 + 4\bar{r} \cdot (1,1,1) - 13 = 0$ and $|\bar{r}|^2 - 2\bar{r} \cdot (10,18,7) + 73 = 0$ touches externally and also find its point of contact.

Que.4(A) Answer any one. (05)

(1) Find equation of cylinder whose generator is parallel to $\frac{x}{3} = \frac{y}{1} = \frac{z}{-2}$ and passing through $y^2 + 2z^2 = 1; x = 3$.

(2) Define: Right Circular cylinder. Find equation of right circular cylinder with axis $\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n}$ and radius r.

Que.4(B) Answer any one. (05)

(1) Find equation of cylinder whose generator is parallel to X-axis and touching the sphere having center $(1,2,0)$ and radius 3.

(2) Find equation of right circular cylinder whose radius is 4 and axis is $x = 2\ y = -z$.

Que.5(A) Answer any one. (05)

(1) State: Rolle's theorem and verify it for the function

$$f(x) = \frac{(x-1)(x-2)}{e^x}\ \forall x \in [1,2].$$

(2) Evaluate: (i) $\lim_{X\to 0} \frac{logx^2}{cotx^2}$

(ii) $\lim_{x\to 1} \left(\frac{1}{logx} - \frac{1}{x-1}\right)$

Que.5(B) Answer any one. (05)

(1) Obtain the equation of sphere having the circle
$x^2 + y^2 + z^2 + 10y - 4z - 8 = 0, x + y + z = 3$ as the great circle.

(2) Derive the equation of a cylinder of which generator remain parallel to the line $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$ and passing through a guiding curve
$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0; z = 0.$

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